Worcester County Mathematics League

Varsity Meet 3 – January 26, 2011

Round 1: Similarity and Pythagorean Theorem

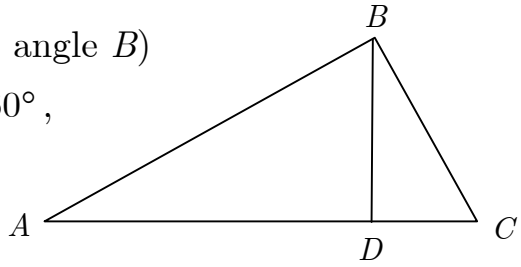
1

All answers must be in simplest exact form in the answer section

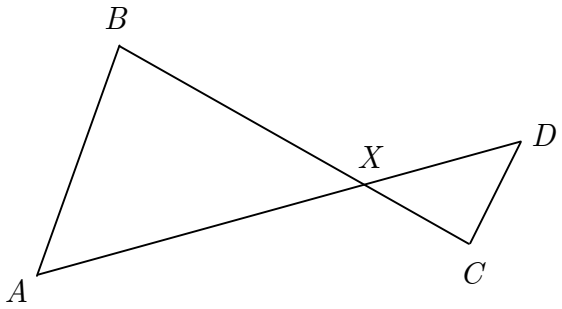
NO CALCULATOR ALLOWED

Note that the diagrams are not drawn to scale

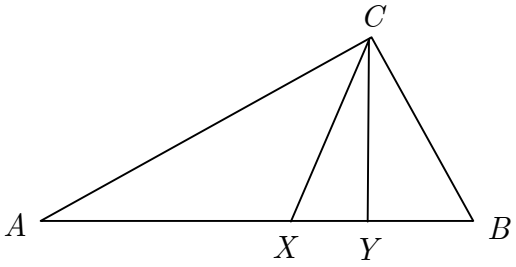
1. The diagram shows right triangle ABC (with right angle B) along with altitude BD. If $BD = 2\sqrt{3}$ and $\angle C = 60°$, find the exact length of $\overline{AD}$.



2. In the diagram below, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \perp \overline{CD}$. If the lengths of $\overline{BC}$, $\overline{AB}$, and $\overline{CD}$ are 12, 4, and 1, respectively, compute the length of $\overline{AD}$.



3. The diagram below shows right triangle ABC with right angle C along with altitude CY. Also, $\overline{CX}$ bisects $\angle ACB$. If $CY = 24$ and $BC = 30$, find the exact length of $\overline{XY}$.



ANSWERS

(1 pt.) 1._____

(2 pts.) 2. _____

(3 pts.) 3._____

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Round 2: Algebra 1 - Open

2

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. Lisa rode her snowmobile 2 miles on January 1st. On each subsequent day she rode 1.5 miles farther than the day before. On what date (month and day), will she have snowmobiled more than 30 miles in a single day?

2. Mike and Ike together saved $550. Mike decided to spend 60% of his savings and Ike decided to spend 40% of his savings. If they were left with a total of $270, how much did Mike spend (in dollars)?

3. When a certain three-digit number is divided by the number with its digits reversed, the quotient is 2 with a remainder of 25. In the original number, the tens' digit is one less than twice the sum of the hundreds' digit and the units' digit. Also, in the original number, if the units' digit is subtracted from the tens' digit, the result is twice the hundreds' digit. What is the original three-digit number?

ANSWERS

(1 pt.) 1._____

(2 pts.) 2. $_____

(3 pts.) 3._____

Burncoat, Bromfield, Clinton

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Round 3: Functions

3

All answers must be placed in the answer section at the bottom

NO CALCULATOR ALLOWED

1. Let $f(x) = \sqrt{2x-1} - \sqrt{17-2x}$. If x and $f(x)$ represent real numbers, how many integers are in the domain of $f(x)$?

2. If $f(x) = 2-3x$ and $g(x) = \dfrac{x-2}{4}$, determine $\left(f \circ g^{-1}\right)(x)$.

3. *Ackermann's function* is defined for non-negative integers n and k by the following:

   i) $f(0,n) = n+1$

   ii) $f(k,0) = f(k-1,1)$, and

   iii) $f(k+1,n+1) = f(k, f(k+1,n))$

   Evaluate $f(2,2)$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 4: Combinations and Permutations

4

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. The student council consists of 5 boys and 6 girls. The council wants to create a blood drive sub-committee consisting of 3 boys and 2 girls. How many different sub-committees are possible?

2. Elliot's car seats 3 people in front and 3 in the back. In how many different ways could you arrange 6 occupants if Elliot must drive and Roberta, his girlfriend, must sit in the front?

3. How many distinguishable permutations of the word CONGRESS do not have the two S's next to each other?

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Worcester County Mathematics League

Varsity Meet 3 – January 26, 2011

Round 5: Analytic Geometry

5

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. Find the value of k so that the lines $5x + 3y = 4$ and $2kx - 5y = 10$ are perpendicular.

2. Let a, b and c be pairwise relatively prime integers. The parabola $y = ax^2 + bx + c$ passes through the point (5, 2) and has a maximum at the point (4, 3). Find the ordered triple (a, b, c).

3. A circle whose center is in the first quadrant is tangent to the x-axis and to the line $y = x$. If the radius of the circle is 4, find the x-coordinate of the center of the circle in simplest radical form.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2. (_____, _____, _____)

(3 pts.) 3. _____

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Varsity Meet 3 – January 26, 2011

TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places, unless stated otherwise (2 pts. each)

APPROVED CALCULATORS ALLOWED

1. Rectangle $ABCD$ has sides of length 7 and 24. What is the shortest distance from point A to diagonal $\overline{BD}$?

2. Consider the following system of inequalities: $x > 0$, $y \geq 0$, and $5x + 3y < 15$. How many ordered pair solutions of this system have only integer coordinates?

3. You would like to name your new dog, but you want to come up with something creative. How many different names can you make, subject to the following four rules?
   Rule #1: The name must be five letters long.
   Rule #2: The name can start with a vowel or a consonant.
   Rule #3: Vowels and consonants must alternate (for simplicity, always consider the letter Y to be a consonant)
   Rule #4: You do not want to name him Rover.

4. The circle with radius 1 and center (2, 1) has two tangent lines that pass through the origin. One of these tangent lines is $y = 0$. Find the slope of the other tangent line.

5. The number $N = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}}$ converges to a real number. Compute N.

6. In a farmer's barn there are some pigs and some chickens (there are at least one of each). If there are a total of 84 legs in the barn, find the sum of all of the possible numbers of chickens.

7. Find the sum of all of the integers between 60 and 70 that are factors of the number $2^{48} - 1$.

8. If $i = \sqrt{-1}$, simplify the following sum to $a + bi$ form:
$$(i + 2i) + (2i^2 + 2i^2) + (3i^3 + 2i^3) + (4i^4 + 2i^4) + (5i^5 + 2i^5) + \ldots + (101i^{101} + 2i^{101})$$

9. If $f(x) = \dfrac{\left(2 - \dfrac{3}{x}\right)^2}{4 - \dfrac{9}{x^2}}$, then $g(t) = \dfrac{f\left(\frac{5}{2} + t\right) - f\left(\frac{5}{2}\right)}{t}$ can be written in reduced form as $\dfrac{3}{at + b}$, where a and b are integers. Find the sum $a + b$.

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ANSWERS

Round 1

1. 6

2. 13

3. $\frac{24}{7} = 3\frac{3}{7} = 3.\overline{428571}$

Round 2

1. January 20 OR Jan 20 OR 1/20

2. 180

3. 371

Round 3

1. 8

2. $-12x - 4$ OR $-4(3x + 1)$
   (or equivalent)

3. 7

Round 4

1. 150

2. 48

3. 15120

Round 5

1. $\frac{3}{2} = 1\frac{1}{2} = 1.5$

2. $(-1, 8, -13)$

3. $4 + 4\sqrt{2} = 4(1 + \sqrt{2})$
   (or equivalent)

Team Round

1. $\frac{168}{25} = 6\frac{18}{25} = 6.72$

2. 6

3. 286,649
   (comma not necessary)

4. $\frac{4}{3} = 1.\bar{3} \approx 1.333$

5. 4

6. 420

7. 128

8. $50 + 53i$

9. 20